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THE MATHEMATICS TEACHER

VOLUME XVIII DECEMBER, 1925 NUMBER 8

AN APPROACH TO THE SOLUTION OF THE PROBLEMS WHICH THE TRADITIONAL MATHEMATICS PROGRAM PRESENTS

By CHARLES A. STONE
University High School, University of Chicago.

Experimentation in the field of education at the present time is causing an agitation toward a revision of the theory of mental discipline and a change in the conception of the cultural values of traditional curricular subjects. Newly formulated principles of psychology, pedagogy, and sociology are affecting radical changes in the educational system.

Reformers in the field of mathematics have come under this influence and are thus working along rational lines. They are evolving a new type of training which is free from the rigidity of the traditional system, and which correlates the various branches of mathematics. Thus they say with Prof. Bobbitt that mathematics should be primarily not a matter of solving difficult problems, but rather a matter of continuously viewing for many years the quantitative aspect of things, and of thinking in accurate terms. The department of mathematics, therefore is to be a department of civics, economics, practical arts, geography, physics, chemistry, etc., in short a department that deals with the things related to life. What is suggested here is a diminution of attention to quantity in the abstract and greatly increased attention to quantity in the concrete.

In support of the new program some of the deficiencies in the old program may be pointed out. The inflexible nature of the traditional arithmetic is such that it favors the maladjustment of those children who differ from the so-called average class. No variafion is permitted even in the average type; and so the time has come when the proper functioning of mathematics is merely accidental. In the upper grades much time has been wasted in maintaining unnecessary skill in the fundamental processes and in dealing with intricacies in problems and processes, many of which are unessential and beyond the comprehension of the child. There has been no attempt to prepare the child for the future abstract type of thinking which the formal treatment of

algebra requires. The children who master the new type of thinking probably do so at the expense of an excessive amount of energy. Those who fail, feel themselves eliminated, and the result is a high pupil mortality while some linger on and continue maladjusted to the detriment of the class morale. These evils should be remedied and one cannot help but feel that this is the duty of the present generation.

The aims of progressive curriculum makers in formulating the mathematics program are as follows:

1. To select material which is worth while in itself and of educational value.

2. To select material which satisfies the real needs and the life and studies of the adolescent boy or girl, and must lie within their experiences. All material not adapted to their mental development and capacities is to be rejected. It should contribute to the pupils' liberal education and not be entirely vocational. It should contribute to his mental training by developing mathematical methods of thought, by teaching him to think accurately and quantitatively and by helping him to acquire effective habits of study as applied in mathematical situations.

3. To bring in fundamental elements of vocational mathematics and those of future mathematical courses.

4. To arrange for tryout courses which will reveal to the pupil his interest, aptitudes and abilities and assist him to decide whether or not he should continue to study mathematics.

5. To obtain equality of opportunity by providing relative amounts of work to take care of individual differences.

There is much discussion about what should be included in the field of Junior High School Mathematics. Traditional material fails to measure up to the test of social worth. A thorough investigation of the practical uses of mathematics in daily life, and of the physical, social and economic interests of the child, should form the basis of such a choice. A fair amount of speculation in the world of tomorrow must be permitted even though the factors in life are of such a variable nature that one cannot definitely determine the future value of present activities.

The "reorganization movement favors the introduction of better teaching methods." These methods should be rationalized (1) through a psychological arrangement of the material and (2) through a study of the learning processes. As suggested by

Prof. Breslich,[1] the material should be arranged in units in psychological order. These units should consist of material that can be conveniently taught together and should be developed in steps of progressive difficulties. They should be short and complete; yet each should bear a significant relation to the whole. Furthermore the work should be put into such farm that an inexperienced teacher can successfully present it. The teacher, however, must have a knowledge of the mental processes which are called into play by the child in his attempt to understand the nature of that which he is required to do.

The organization should begin with the seventh year work and should consist of an abundance of concrete material, as a very small percentage of adolescent boys and girls are able to think in the abstract. Algebra should be introduced as a natural means of expressing facts about number. One cannot proceed very far in the study of mathematics without encountering the need for algebraic symbolism and formulas, for ability to solve equations and to evaluate algebraic expressions. When the child is able to appreciate the convenience of symbolic language in stating the conditions of a problem, algebra will have a significant meaning for him. Concrete geometry should also be taught through actual measuring and other sensory processes, because space is a complex form of relational experience. It is not to be the demonstrative geometry of the senior high school, but it is essentially the study of the most familiar facts in human experience easily comprehended by the immature mind. It is experience in getting space relations, the geometry of the class room, the home, the park, the city and the field. The pupils measure distances, angles, areas and feel that they are studying real things. Numerical trigonometry is to be introduced for the purpose of making indirect measurement. To be sure, work in arithmetic is not to be omitted for it must continue for several years after the sixth grade, but the learning of the processes should cease to be mainly a drill undertaking. It should be planned with the utmost care and kept up with systematic regularity throughout the whole course. The algebra and geometry should be correlated and the arithmetic should be incidental to both. Thus the three should

[1]Breslich, E. R.: *The Unitary Organization of the Mathematics of the Seventh, Eighth and Ninth Grades.* THE MATHEMATICS TEACHER, April, 1923, Vol. XVI, No. 4.

be taught together. This plan makes possible the choice of processes for the solution of a particular difficulty.

As a means of improving instruction and facilitating the teaching of the materials as suggested, supervised study should be introduced for it gives the teacher a chance to discover what aspects of work need emphasis. It makes it possible to clear up individual difficulties and to direct the most economical mode of proceeding with a task. Bad habits of study are checked before they become permanent, and desirable habits are started and encouraged. Thus it is possible, with the aid of supervision and use of texts which contain concrete material and which are suitable for the supervised study period, to develop the ability to apply the information to particular situations. As Prof. Judd points out, the schools are remiss in that phase of mental life called application. He further states that the mental processes involved in acquiring skill in algebra and geometry differ in character from the mental processes involved in applying this mathematics to physics and practical situations in the shop. Thus it is up to the school to furnish the kinds of situations which cultivate this kind of power. This can be done by presenting to the child problems which will furnish opportunity for their use.

Another aim of teaching mathematics is to develop the habit of thinking accurately in quantitative terms, and of seeing things in quantitative ways. The child is not always able to see immediately the functional relationship which is expressed in the formula $i = prt$. He may be able to manipulate the symbols in order to obtain the result without any knowledge of the interdependence of the variables which express the formula. By continuously viewing throughout the junior high school years, the quantitative aspects are such as are met in thousands of concrete situations, a familiarity with quantitative relationship may be cultivated. Considerable guidance brought about only through the use of text books which take the new ideas into consideration is needed to bring about this mental attitude. The mental condition of our population proves the fallacy of the assumption that the study of the conceptive quantities and quantitative relations in the abstract will in some magical way give one the power to see the quantitive aspects of everything.

The socialized recitation will also have to be given a place in the new scheme. Thus there must be a complete change in the

conduct of the recitation. The teacher has been doing too much of the work and the pupils have been trained to dependence and inactivity, rather than to independence and to self activity. This means that in the future the teachers must be less prominent and the pupils more prominent, the teacher contributing less and the children more. After the pupils have learned thoroughly during the supervised study period they should be able and should be given the opportunity to talk on topics without help or hindrance from the teacher. This will aid them in organizing their thoughts. One of the most important functions of recitation period is the development of initiative and self-reliance. These qualities are fundamental and lie at the very basis of a democracy such as ours. To be successful one must have nerve and decision to act quickly and assume initiative in terms of energy. To develop initiative, the pupil must be given an opportunity to exercise it.

No one will deny that the adolescent child must be furnished with a wide range of experience in all his activities so as to broaden his outlook on a newly conscious life. The work in mathematics must therefore offer an opportunity to explore the possibilities of the field. A general course best suits this purpose and should contain the kind of information which will assist the child in gaining an insight into the more general aspects of education. The 7th grade is the most economical place for the child to discover his aptitudes for the various activities of life, and the school is performing its function only when it assists in the direction and selection of these activities. All classes of children profit by a general survey of mathematics. Those who are likely to make a specialty of it are assured of the suitability of their choice. Those who realize their limitations and weaknesses will have no regrets. Those who have a moderate use for it will have been benefited by a general view of the whole subject. This general course in the 7th grade should be a complete unit in itself in order to do justice to those who proceed no further. The work must articulate above as well as below, so as not to penalize those who continue the study of the subject. It must furnish equal opportunity for all to be worthy of a place in a democratic system.

The work of the 8th grade should continue the treatment of graphs, equations and formulas. The material will be largely

algebra of the concrete type and will be correlated with the arithmetic of the home, community, school and industry. The 9th grade course will continue the algebra and some time may be devoted to geometric relations in a more formal way. Business methods are to be discussed through the introduction of suitable arithmetic.

The program as outlined may be a radical departure from the traditional course in mathematics, but the greatest justification seems to be in the increased holding power of the schools as is evidenced where the new plan of education is being worked out. This will likwise counteract the present tendency in many states to drop mathematics from the curriculum of required subjects.

TEACHING MATHEMATICS TO GIRLS[1]

By JAMES BYRNIE SHAW
The University of Illinois

No doubt the title of this paper suggests a question at once to the mind of the reader: Why girls? It is, however, easy to discover that among the girls who enter freshman classes there is a rather widespread idea that Mathematics is a very difficult subject, and that one should take as little as possible and meet the requirements for graduation. Owing to early inadequate teaching, and to lack of understanding the meaning of the elementary parts of mathematics, a large part of the entering class in any university or college, will come in with a dread of the subject, and sometimes a loathing for it, which will interfere seriously with success in passing a freshman course; and, what is of more serious import, will pass on to succeeding generations the idea that it is a branch of the curriculum of little interest, and to be learned only with an extraordinary amount of work and worry. To those of us who know the fascination of this field of human thought, this should be a matter of keen regret, but we must assume the blame ourselves. In recent times the value of Mathematics for its applications to Engineering, Chemistry, Physics, and the like, has been much over-stressed; so much so that recent text books would lead a casual observer to the conclusion that the only object of the mathematician was to serve as a handmaid for the engineering profession. In the effort to make the subject concrete and interesting, largely to boys, we have forgotten some very fundamental values of mathematics. For it has values of a deep human interest beyond its mere utility. That mathematicians have been content to leave the situation unchanged simply shows how little we consider these values, or how indifferent we are as to what the public thinks of our work. Usually no account is taken of the appeal that mathematics should make as a fine art, an art whose constructions are as fascinating as music; no account is taken of the philosophical character it has, as a branch of human thought with continuous growth for centuries, perma-

[1] Read before the Illinois section of the Mathematical Association of America, at Peoria, May 8, 1925.

nent in character, its investigations dominating the advance of thought, its results penetrating the profoundest questions of philosophy. Compared with the values mathematics possesses of a philosophic and an artistic character, its utilitarian values sink into insignificance. The answers it gives to such questions as: What is reality? What may we know by intellectual processes? What is proof? What is certainty? What is truth? mean more in the end for civilization than a few more solutions for engineering differential equations, or the determination of a few orbits of electrons.

What then are the difficulties in teaching mathematics to girls? Are they at all different from the difficulties boys have? Should the teaching methods differ? Should classes be segregated in order to make it possible to use different methods?

We will all agree that the student taught is one of the necessary factors in the teaching process. On the other hand the subject taught is also a necessary factor. There is further the teacher whose function it is to bring the first two factors into a union. But the teacher is not a necessary factor. I mean that students learn without a teacher, and sometimes in spite of the teacher. Quite often the only thing the instructor in charge of a class does is to assign tasks and record grades. This is a sweatshop method of handling classes and of course is not to be dignified with the title of teaching.

I am also quite well aware that many educationists put forward the thesis that there is no difference in the rate of learning of boys and of girls, nor in the type of learning. This is often asserted upon the basis of so-called tests. I shall not quarrel with these statements, but merely suggest that it is doubtful if they test the essential factor in the problem. At least my own observation leads me to a conclusion diametrically opposed to the one stated. If boys and girls are taught alike there is a difference in the results obtained, and if girls are taught in the proper way the difference in results is enormous. These differences arise from the psychological natures of girls and boys. They may not be so evident in early grades, but they begin to appear in secondary and college classes. The chief reason they are not well-known is that practically all Mathematics teaching is done much as it has been done for centuries, and with men in

view as the students. The girls have got what they could. Even girls' colleges are not different in most cases from men's in this respect.

Let us emphasize first the fact that girls are more highly impressionable than boys. They respond very much more easily to suggestions than do boys. They are very sensitive to the atmosphere of the class-room, and will be depressed or stimulated by it. What constitutes atmosphere is intangible and invisible, difficult to analyze out, but it is enlivening or deadly nevertheless. It is this that makes a girl remark: "I simply cannot work for that instructor. I am paralyzed whenever he calls on me." Or sometimes they say: "He gets me so nervous in his class that I am worn out when it is over." Of course the same things are met in other classes than those in Mathematics. In the same list of adverse conditions we may put those arising from the mistaken notion some teachers have that the Socratic method of leading a student to arrive at the truth by expert questioning and forcing him to see the absurdity of statements he has made, is the same thing as humiliating a student by ridicule and sarcasm in front of a class. This of course is fatal with girls; for they are proud, and whether consciously or not, they resent naturally anything that makes them appear at a disadvantage, and they fear a class where they may be helpless against this sort of teaching. This is one good reason for segregating classes, for failure before the other sex is humiliating.[1]

Girls are interested in persons, and not much in things or in abstractions. They are not naturally enthusiastic over scientific advances, for instance. If they show an interest in these it is usually because they are interested in some person who in his turn is interested in science. This fact makes it possible for girls to be interested in literature and history, where the personal element enters, even if the characters are fictitious. They can find some human attachments in history or literature; but in physics, chemistry, and mathematics, there is a sort of impersonality which is not attractive to girls. Girls are not interested in things as are boys, consequently the applications of Mathematics to things does not appeal to them. They must see Mathe-

[1] Further, girls hesitate to ask questions, which may make them appear stupid, or which provoke ridicule, consequently fail to have the subject well-explained and in classes of girls unfailing courtesy on the part of the teacher is a necessity.

matics as entering life itself in some way to be interested in it. For this reason, while girls like home-work assignments, they are not interested in merely solving problems. The question they ask is: "Where does the solving of quadratics enter my life as a vitalizing factor?" The use of quadratics in working out maximal stresses and strains is of little importance in their scale of values. But when quadratics can be attached to as interesting personalities as Diophantos, or Omar the Tentmaker, they come to have value.

Owing to the long dominance of women by men, girls are unconsciously accustomed to feel the weight of public opinion and to yield to it. When they hear from early childhood the often repeated statement that Mathematics is hard, uninteresting, and is mainly a man's subject, that women never have done anything in it, that all girls dread it, they are impressed by the mere repetition of this fallacy, and are depressed and even terrified when face to face with required courses in Mathematics. Besides this, girls aspire to excel in what they undertake, and they are loath to gamble with a subject in which they stand many chances to lose, or at best, to make low grades. If this fear and dread alone were removed there would be a great change in the attitude of girls toward Mathematics. Records show that on the whole grades are higher for girls than for boys even under teaching designed primarily for boys. Under teaching fitted to girls' classes the difference is very large, as facts show. It should be incumbent upon every teacher of Mathematics to undertake a propaganda campaign to change the current notions in this regard.

Girls unconsciously resent being treated impersonally. They feel they are something more than merely the occupant of seat number A-X. Each girl feels that she is an individual, different from other people, and she expects to be treated as different. The wise teacher in any subject will take this into account and will show an interest in the personal individual accomplishment of each girl. This alone often will change failure into success. When a girl says with regard to an instructor to whom she has gone for help in understanding the subject: "He merely laughed at me," we cannot fail to see why failure is inevitable. The teacher perhaps thought it wise to throw her on her own resources, but she interpreted his remark to mean indifference as

to whether she got the course or not. When an instructor remarks at the beginning of a course: "This is the subject the department flunks the highest percentage of students in, and I flunk more than anyone else. Get to work," he has already lowered the grade of every girl one letter.

These items and similar ones bear upon the handling of classes of girls, whatever the subject. But there are some things peculiar to Mathematics which we must note. For instance there are certain special characters in the thinking of girls which are of immense importance in teaching Mathematics, and should determine the form of presentation of the topics considered. The first of these is the synthetic and highly imaginative character prominent in the thinking of girls. A girl is not naturally analytic. She does not order her life by deductive reasoning. If she does not *feel* the answer to a problem, cannot perceive directly a line of conduct, she is not likely to get much out of a solution produced as the conclusion of a chain of deductions. She may accept it and go ahead blindly, but it does not become part of her. The imaginative character of feminine processes of inference is a great advantage in Mathematics, for most of the work in Mathematics is based upon imagination. As a matter of tested fact, if Mathematics is presented largely in an imaginative way, in the same way that mathematical discoveries are made, the girl's mind will very readily absorb an astonishing amount of it. Even a third more can be done in a semester by such methods than in other classes conducted in the usual way. The effort must be made to present the subject intuitively. The much disputed intuitive character of feminine inference has been denied, it has been called a superior form of intelligence, it has been called merely a different type of association, it has been explained as an imaginative perception, it has been called penetrative insight; but whatever the psychologist pleases to do with it, it is a fact that everyone who understands girls reckons with. The intuitive method of presentation has been that of such famous lecturers on Mathematics as Klein, and Sylvester, and others, so it has the weight of great names back of it, but it does not have to be confined to graduate courses. When the different topics of an elementary course are thought over, it is surprising how far they may be reduced to very obvious data. And we need not fear that the fancied value Mathematics has in developing the

reasoning power will be lost by using intuitive methods. Plenty of inference of deductive type will be left.

Girls excel boys in observation, consequently any method of presentation which will enable them to see relations visually will appeal to them and they will learn them readily. The arrangement of work so as to make its schematic character carry some of the burden will help. Girls may be taught to read geometric diagrams almost as fast as print, and algebraic forms can be treated in a similar manner. We all are aware of the great advantage of writing the elements of a determinant in a square array, but there are many other forms that are also useful. Graphic work should be made prominent for the same reason.

The feminine mind makes concrete or particular cases serve as types for general cases. We might almost say that reasoning by metaphors is much more common than reasoning by abstractions. This makes it possible to take special well-chosen instances of mathematical theorems and develop the subject directly from them. I do not mean that general theorems are not introduced in the work, but that they are not *FIRST* introduced. The special example should be chosen so as to exhibit every feature of the theorem under consideration. Others should follow till the essential ideas are apprehended, and can be used, then a general statement or summing up should follow. For instance, the usual text-book works out the completion of the square in solving quadratics, and states the result as a formula, and stops with this. The girl will see that this method works, but it is purely a method of doing a trick, and does not mean much. If however, it is shown first that the simple method of factoring an expression, and using the arithmetical principle that when a product of factors is zero, at least one of them is zero, enables us to take the expression apart, as it were; that if we can reduce a quadratic problem to the difference of two squares we can factor it, and that this can be done by the process of completing the square provided we can set the residual term equal to a square; then we have made it evident why we must complete the square. But also it is seen that an auxiliary equation is necessary, which is in fact a simple case of a Galois resolvent, and we are getting into the heart of the problem. It becomes evident why we must increase our domain of number by square roots. Also it is evident

we must invent a new type of number, the complex number, in order to handle quadratics. Again, by building up the quadratic as the product of two factors, it is easy to present the sum and the product of the roots in terms of the coefficients; and by trying to solve the quadratic from the elementary symmetric functions, Lagrange's ideas appear, and the necessity of finding some other function, unsymmetric in the roots, is evident. Thence the formula for the roots takes a new meaning, for its rational part is the half sum of the roots and its radical part the half difference of the roots. All this comes out of the particular cases considered, and is easily shown then to be general in its application.

Girls are interested in persons, and this interest may be utilized by having short biographies of prominent mathematicians. What I ask of my classes is a 200 word theme setting forth what they have found interesting in the life of the mathematician assigned. The most common remark is the surprise of the student to find the life of a mathematician interesting at all. I have had many papers expressing the delight and the real pleasure the student found in reading about Descartes, Galois, Kovalevsky, Sylvester, and several others. Surprise is always expressed that mathematicians are interested in music and the arts, and so many other human things. Should it not make us blush a little that anyone should be surprised to find that mathematicians are human? The trouble of course is that in courses in literature, history, art, and the like, the great leaders are spoken of, and become real persons; while in mathematics courses little is ever said about the great names of the subject, many of whom have been the very highest leaders of human thought and progress.

It follows also that the instructor should undertake to be interesting and have something to say about life in general, and especially those features of it which are connected with mathematical things. An instance is the subject of mathematical induction, which can be made to serve as a text for a general explanation of deduction, scientific induction, and reasoning in general. I do not hesitate indeed to bring in questions of proof of religion, the meaning of faith, and many other such topics.

Let us pass now to the other factor of the problem, the subject itself. The history of mathematics shows that it is a highly

synthetic subject, steadily growing, not only by addition to what is already established, but by new branches. In the long course of the centuries it is in fact the only subject of human knowledge in which we find anything like permanent results in fundamental principles. The present swirl in scientific theories, with the most cherished principles and laws passing away in a whirl of dust, should make it evident to anyone that a subject which remains unchanged is certainly of most vital interest. One of my students said: "It is a relief to find there is at least one thing which I study which is not going to be upset next year." It seems regrettable that we turn out year by year thousands of students in the various branches of mathematics with no conception whatever of the magnificence of this structure of the ages, who look upon mathematics as merely a big computing machine. The first thing I undertake therefore with a class of girls is to give them a glimpse of the grandeur it possesses. This means that the philosophic aspects of it are brought out early. The attempt of Pythagoras to explain the universe by integers,—a dream almost realizable nowadays—but confronted with the difficulty of the irrational; the insistence of Plato that Mathematics is necessary for entry into philosophy—realized today by the most advanced school,—since in Mathematics we are compelled to make use of the pure intelligence to study the suprasensual, and find there entities which are eternal and universal; the fundamental notion of Descartes that we must above all think clearly, basing our arugments upon what is directly obvious, and that so far as one thinks he really lives; even Omar Khayyam's poetic demand that from a mathematician's point of view we must treat data of life as they are, rather than as they may be or as we regret they have been;—does anyone think these do not appeal to the interest of the twentieth century girl? Let him try it and be convinced.

On the side of art, if we follow up the idea of the Galois resolvent equation with its roots each expressible rationally in terms of any one, and the rational forms thus arising forming a group, which in its turn may be studied as a new entity, and is seen to explain the meaning of symmetry, as in a crystal; or the many varieties of algebraic form which become almost architectural in character as they are put together; or the invariancies that appear in elementary expressions, and run like golden

threads through whole chains of expressions;—if we take advantage of all that any branch of mathematics offers us, we can easily exhibit it as a fine art, which handles ethereal substance it is true, but is none the less capable of becoming silent music.

But I have heard the objection made: "Ah! I see that all you teach is a mass of glittering generalities." I think of course the person who made this remark missed the point; but upon the whole is not Mathematics a collection of generalities that not only glitter but shine with a steady brilliancy? I am glad to be able to open the eyes of students to the illumination on thought and life that is to be found here. If the alternative is that classwork is to be confined to an accumulation of details and the manipulation of symbols and figures, if the goal is merely the acquiring of a clever ability to solve problems, if the fundamental principle is something like that which used to assert that before a student could begin to appreciate a foreign literature he must have acquired a perfect knowledge of grammar, then I am certainly on the side of the generalities. To be merely expert at these other things is about as useful as to learn to throw a boomerang, to write one's name on the ice, or to be clever at sleight-of-hand. Not that these abilities do not have value, but that in the time we have at our disposal there are many things much more essential for the student's progress in mathematics. Moreover my experience with girls shows that when they understand the real essence of the subject the technique will take care of itslf to a considerable extent. I have never found any who could not learn to do easily what she well understood. But I would go further and assert that much of the technique of expressions in the average textbook is very clumsy and not readable, or understandable, by the student. It is a source of surprise to me frequently that graduate students and teachers of mathematics really have so little skilful technique. I find this deficiency is due usually to a lack of full comprehension of the fundamental nature of the subject owing to a limited reading acquaintance with mathematics. If the topics studied are presented as the natural flowering of the ideas at their root, the student will learn not only the facts of mathematics but will acquire some notion of how to develop mathematics herself. If mathematics is a fine art, then every student who aspires to become mathematical must learn how to paint her own pictures.

It has been said by an eminent authority[1] that a college course is of little importance to a girl in developing her own native methods of thinking, and may even be a detriment. It would follow that there is sufficient ground here for urging that courses in such subjects as can develop the native ability of girls should be made to do this so far as possible. I am willing to assert that one of these subjects is Mathematics. But it must be taught in such a way as to bring out the imaginative ability, the inventive ability, the creative ability of girls. It should be made to replace cold logic by ideals flushed with warmth, to appeal to all the esthetic sense. It should give a place to spontaneity and should avoid rules and mechanical procedures. This can be accomplished by putting girls into their own classes where the subject may be so organized as to give play to just these abilities. No one need fear that there will be any loss to Mathematics or to mathematical training in doing this, for my experience leads me to conclude that there is on the other hand a great gain. The best test will be their ability to understand, to follow, and to delight in courses in mathematics that come later.

We stand at the entrance to many new things today and among them let us hope there is a new valuation of Mathematics as a humanizing culture, particularly adapted to give poise in a dizzy existence, to keep strong the flame of truth, to appeal specially to the twentieth century girl as a fascinating branch of study. Let us hope that in its flowering she may find self-confidence, standards by which to evaluate all learning, a chance to excel, a penetrating insight into the eternal realities, a firm foundation for all she discovers by her intuitive methods, a respect for the pioneers of thought, and a vision of the Beauty which shall never perish.

[1] Gina Lombroso, *The Soul of Woman.*

SCALES OF NOTATION

By GORDON R. MIRICK and VERA SANFORD
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The inclusion of the topic *Scales of Notation* in the College Board's requirements in Advanced Algebra is justified not for the intrinsic usefulness of the subject for its practical value is nil, but solely for the insight that it gives into the fortuitous nature of our number system.

The worth of the subject lies in seeing whether one can separate the general properties of the place value notation from the specific properties of the decimal system, and in seeing also whether one has sufficient strength of mind to follow the logical consequences of a different number base without reverting to the one that familiarity makes us take for granted. In this way, it is kin to the geometries based on postulates that differ from Euclid's. Both emphasize the arbitrary nature of our definitions and postulates. For this reason, both are of value.

Like other interesting by-paths in mathematics, this topic appears only rarely in text books[1] and so a brief discussion of it may serve as a guide to the subject. The discussion does not pretend to be detailed or scientific; it only claims to select a few vantage points along the by-path.

Our number system is built on the idea of *place value*. In the number 2222, for instance, the first 2 stands for $2 \cdot 1000$, the second for $2 \cdot 100$, the third for $2 \cdot 10$ and the last for 2 units. This could be written $2 \cdot 10^3 + 2 \cdot 10^2 + 2 \cdot 10 + 2$. The 10 is called the *base* or *radix* of the number system. If we let r stand for the base, this number becomes $2r^3 + 2r^2 + 2r + 2$.

The choice of 10 for the radix of our number system is doubtless due to the use of one's fingers and thumbs in counting. The fact that systems on other bases are found among certain primi-

[1] A detailed discussion of the subject appears in Siceloff and Smith *College Algebra*, Ginn, 1924, pp. 239-246. In Bourlet *Cours d' Algebre* the subject enters incidentally in problems on geometric progressions. E. Grandgaignage et L. Reclus *Arithmetique, Paris* (1920) discusses it briefly under notation.

tive people[1] is interesting but the traces of other systems in our number names are more important. A system based on 20 appears in our counting by scores[2]. We have words connected with a possible scale of 12 in our dozen and gross, and hints of it in the division of the foot into twelve inches and the old-time twelve ounce pound.

Another base might have been chosen and in some respects the base 12 would have been more convenient. To see how such systems would operate and to investigate their similarities to and differences from our system, let us use $a, b, c, \ldots$ to represent the digits of a number, let r be the base and let N_r mean the number expressed in terms of the base r. Then $N_r = a + br + cr^2 + dr^3 \ldots$ where none of the quantities $a, b, c, \ldots$ can be equal to or greater than r. It appears then that the digits required for the system are $0, 1, 2, 3 \ldots, r\text{-}1$. It also follows that no useful number system could be built on the base *1*.

As a preliminary step, let us study two number systems: the binary (base 2), and the duodecimal (base 12). In the first case, the digits 0 and 1 suffice. In the second, we need a digit for 10, let us say•, and one for eleven ≠. Evidently as the value of the base increases, the number of digits required increases and the numbers become more compact.

Decimal	*Binary*	*Duodecimal*
0	0	0
1	1	1
$2 = 2$	10	2
$3 = 1 + 2$	11	3
$4 = 2^2$	100	4
$8 = 2^3$	1000	8
$10 = 2^3 + 2$	1010	•
11	1011	≠
12	1100	10
$16 = 2^4 = 12 + 4$	10000	14
24	11000	20
144		100
1728		1000

[1] D. E. Smith, *History of Mathematics*, Vol. I, pp. 8-14.
[2] cf the French 'quatre-vingt' = 80, literally "four twenties."

In expressing numbers with different scales of notation, three types of problems arise:

I. Given a number in the decimal system, express that same number in another system whose base is known.

II. Given a number in a given system, find the equivalent number in the decimal system.

III. Given a number in the decimal system and the equivalent number in another system, find the base of the second system.

Problem I. To change a number from the decimal system to another system whose base is r.

Since $N_r = a + br + cr^2 + dr^3 \ldots\ldots$., then the first term is the only one which does not contain r as a factor. Accordingly a equals the remainder when N_r is divided by r. The resulting quotient Q_1 equals $b + cr + dr^2 \ldots$. Similarly, b is the remainder when Q_1 is divided by r. This process should be continued until division is no longer possible. The last quotient is the coefficient of the highest power of r in the number N_r.

Examples: Express 215_{10} in the scale of 7.

$$\begin{array}{r} 7)215 \\ 7)30 - 5 = a \\ 4 - 2 = b \\ 4 = c \end{array}$$

$$215_{10} = 425_7$$

Express 14178_{10} in the scale of 12.

$$\begin{array}{r} 12)14178 \\ 12)1181 - 6 = a \\ 12)98 - 5 = b \\ 8 - 2 = c \\ 8 = d \end{array}$$

$$14178_{10} = 8256_{12}$$

Problem II. To change a number in a given scale to the equivalent number in the decimal system.

Since $Nr = a + br + cr^2 + dr^3 \ldots\ldots$ and since $r, a, b, c, d, \ldots\ldots$ are known quantities, the equivalent number in the scale 10 is obtained by substitution.

Examples: Express 5231_6 in the scale of 10.

$$\begin{aligned} 5231_6 &= 1 + 3 \cdot 6 + 2 \cdot 36 + 5 \cdot 216 \\ &= 1 + 18 + 72 + 1080 \\ &= 1171_{10} \end{aligned}$$

Express 567_{12} in the scale of 10.

$$\begin{aligned} 567_{12} &= 7 + 6 \cdot 12 + 5 \cdot 144 \\ &= 7 + 72 + 720 \\ &= 799_{10} \end{aligned}$$

Express $15^*\neq 12$ in the scale of 10.

$$\begin{aligned} 15^*\neq 12 &= \neq + {}^* \cdot 12 + 5 \cdot 144 + 1 \cdot 1728 \\ &= 11 + 120 + 720 + 1728 \\ &= 2579_{10} \end{aligned}$$

Problem III. To find the base of a given number whose equivalent value in the decimal system is known.

In this case we are given $N_r = N'_{10}$, where r is to be found. We can express N_r in the form $a + br + cr^2 + dr^3$ and thus by equating this to N'_{10} we have an equation in r where N'_{10}, a, b, c, etc., are known,—

$$N'_{10} = a + br + cr^2 + dr^3 \cdot \ldots\ldots$$

If N_r is a number of two digits we have a linear equation, if N_r has three digits, we have a quadratic: both of which are easily solved. If N_r is a number of more than three digits we have an equation which is most easily solved by the factor theorem; i. e. guess a value for r, and substitute it in the equation. If the equation is satisfied, the correct value has been found, if not try another value. It should be noted that r must be an integer greater than the largest of the digits of N_r.

Examples: Given $1423_r = 375_{10}$

$$1423_r = 3 + 2r + 4r^2 + r^3$$

Guessing 5, $3 + 2\cdot5 + 4\cdot5^2 + 5^3$

$3 + 10 + 100 + 125 = 238_{10}$

Guessing 6, $1423_r = 3 + 2\cdot6 + 4\cdot6^2 + 6^3$

$= 3 + 12 + 144 + 216 = 375_{10}$

$\therefore r = 6.$

Given $1*6r = 237_{10}$

$= 6 + *r + r^2$

Guessing 11, $= 6 + *11 + (11)^2$

$= 6 + 110 + 121 = 237_{10}$

$\therefore r = 11$

Applying these principles to fractions, we are reminded of the sexagesimals (fractions whose denominators were powers of 60) that proved so useful to old-time scientists: the fractions that formed the pattern of our decimals. Suppose we consider the simple case of duodecimal fractions. In this system,

0.1 means $_1{}^1\!/_2$

0.01 means $(_1{}^1\!/_2)^2$

0.001 means $(_1{}^1\!/_2)^3$, etc.

Accordingly $(\frac{1}{2})_{10} = (_1{}^6\!/_2)_{10} = 0.6_{12}$

$(\frac{1}{3})_{10} = 0.4_{12}$

$(\frac{1}{4})_{10} = 0.3_{12}$

$(\frac{1}{6})_{10} = 0.2_{12}$

$(\frac{1}{8})_{10} = 0.16_{12}$

$(\frac{1}{9})_{10} = 0.14_{12}$

$(\frac{1}{5})_{10} = 0.24972497_{12} \ldots\ldots\ldots$

Evidently duodecimals are a "common fraction system," for the fractions that cannot be expressed exactly are the multiples of $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{11}$, $\frac{1}{13}$, $\frac{1}{17}$, etc.

Operations with numbers in scales different from ten are identical with operations with numbers in the scale of ten. But one must remember that a digit in the second place of a number represents a multiple of r and not of 10.

Examples.

Addition	*Subtraction*
263_7	$\not{4}38_{12}$
521_7	$32^{\bullet}9_{12}$
1014_7	$814\not{-}_{12}$

Multiplication	*Division*
562_8	$3_5)432_5$
437_8	124_5
5036	
2126	
2710	
317316_8	

In our decimal arithmetic, we have simple rules for testing divisibility of numbers by 2, 3, 5, 9, 10, 11. How far do these tests depend on the fact that 10 is the base of the system? And what general laws do they represent? For instance the test for divisibility by 2 breaks down in the case of 12_3 whose value is 5_{10}. It can be shown that the other tests do not apply in other cases. Clearly the number $N_r = a + br + cr^2 + \ldots\ldots$ is divisible by r or one of its factors, only when a is zero or a multiple of the factor in question. Tests for divisibility by 2, 5, 10 in our system belong here.

Now if we rearrange our expression for N_r thus:

$N_r = (a + b + c + d \ldots\ldots) + b\ (r - 1) + c\ (r^2 - 1) + d\ (r^3 - 1) \ldots\ldots$ it is clear that each term involving r is divisible by $r - 1$. Then N_r is divisible by $r - 1$, or its integral factors, only when $(a + b + c + d \ldots\ldots)$ has that same number as a factor. (This covers our division by 3 and 9.)

Again, since $N_r = (a - b + c - d \ldots\ldots) + b\ (r + 1) + c\ (r^2 - 1) + d\ (r^3 + 1) \ldots\ldots$ and since each parenthesis involving r has $r + 1$ for a factor, then N' has $r+1$, or its factors, as a factor only when $(a - b + c - d + \ldots\ldots)$ is divisible by this same number. A special case is our test for divisibility by 11.

Since there are simple tests for divisibility by $r - 1$, r, and $r + 1$, and their factors, it is evident that in each of these number systems, there are simple tests for divisibility by 2 and 3.

It would be interesting to speculate which system would be really the most useful one. It is an ideal subject for speculation for one's conclusions will never be upset by seeing the thing tested out in reality. The whole subject has that adventuresome quality that flavors all enterprises where one sets aside habits—bonds if you will—that long practice has made automatic, and attempts, for the time being, to be guided by definitions and postulates of which these well trained bonds illustrate only special cases.

EXERCISES

1. How would you write:

a. 382_{10} using the base 6?
b. 658_{10} " " " 7?
c. 878_{10} " " " 12?
d. 416_{10} " " " 8?

2. How would you write:

a. 3234_6 using the base 10?
b. 1434_6 " " " 10?
c. 3422_5 " " " 10?
d. 1621_7 " " " 10?

3. Find the base in each of the following cases where the equivalent value of each number is given:

a. $612_r = 878_{10}$, $r = ?$
b. $640_r = 416_{10}$, $r = ?$
c. $3422_r = 487_{10}$, $r = ?$
d. $3234_r = 742_{10}$, $r = ?$

4. Perform the following operations, checking each by changing each of the numbers to the base 10.

Add 245_8, 367_8, 102_8

Subtract 3005_9 from 1876_9

Multiply 237_{12} by 185_{12}

Divide 3840_{12} by 30_{12}

THE TEACHING OF CANCELLATION

By JOS. A. NYBERG
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In a recent article (An Analysis of the Teaching of Cancellation in Algebraic Fractions, by August Grossman, Ben Blewett Junior High School, St. Louis, Mo., April, 1924) the analysis of cancellation was divided into four steps:

1. Just what does "cancellation" mean?
2. What sort of cancellation do we employ in fractions?
3. How far does the effect of a factor extend?
4. "When can you cancel?"

The present writer has been able to condense this work into two steps *provided* that, during the earlier work in the course, he has taken precautions against the formation of undesirable habits from other topics.

Mr. Grossman, in his step 1, analyses what cancellation might mean, and shows that in general it implies the undoing of a previous operation. This is doubtless the correct meaning of the word as used in our everyday conversation. If, however, the correct meaning of the word sets up bad habits of work in our algebraic processes, then there is good reason why we should omit any reference to the most general meaning of the word and confine its use chiefly to those concepts that make the word valuable in algebra. Some teachers may solve the difficulty by saying that we should omit the word "cancellation" entirely, just as some teachers omit the use of the word "transposition." Regardless of whether we use the word or not, there are two fundamental concepts about the reduction of fractions which the pupil should learn.

Before stating these two ideas, let us see what are some of the bad habits of thought or action that may interfere with the learning of the correct habits. The first bad habit arises when the teacher changes an equation like

$$5x + 8 = 9 + x + 8 \quad \text{to} \quad 5x = 9 + x$$

and use the word "cancellation" as an explanation of how the number 8 disappeared from the equation. Even if 8 is subtracted

from both numbers of the equation and the left number written as $5x + 8 - 8$, it is still undesirable to say that $+ 8 - 8 = 0$ because $+ 8$ and $- 8$ *cancel* each other. It is better to say that *the sum* of $+ 8$ and $- 8$ is 0, thereby using the same words as when we say that the sum of $+ 4$ and $- 6$ is $- 2$, or the sum of $+ 9$ and $- 6$ is 3. In any work involving addition, the number 0 is not particularly different from any other number.

The pupil may develop other bad habits so strange and startling that the teacher seldom suspects the existence of them. The writer ascribes their formation to the fact that many pupils are more clever than the teacher thinks, and that, after listening to the teacher's explanation of how a problem should be solved, the pupils begin to experiment with other methods with a view to finding an easier method. The writer was surprised one day to learn from a pupil that after all the quickest way to solve the equation[1]

$$\frac{5}{x-6} + \frac{4}{x-9} = \frac{8}{x-7} + \frac{1}{x-10}$$

was to invert each of the fractions, changing the equation to

$$\frac{x-6}{5} + \frac{x-9}{4} = \frac{x-7}{8} + \frac{x-10}{1}$$

To the pupil, this discovery was nothing less than a stroke of genius. To prove that his method was correct he solved both equations and found that $x = 11$ for both equations.

The above equation is an illustration of the formation of a bad habit because the pupil sometimes obtains the correct answer by an incorrect method of his own discovery. Thus, in reducing the fraction $\frac{x\sqrt[2]{} - y\sqrt[2]{}}{x-y}$ we may get the correct answer by dividing $x\sqrt[2]{}$ by x and $- y\sqrt[2]{}$ by $- y$. Such habits can be avoided only by examining carefully the exercises used at the beginning of the work.

Again, regardless of what the teacher may say, when the pupil sees that $\frac{3a + 3b}{3} = a + b$, the pupil is very likely filing away

[1] This is not an artificial illustration by the writer. It is ex. 8, page 52 in a textbook

in his mind for future use a rule like this: Cross out the repeated numbers, and there's the answer! Having adopted this rule (unknown to the teacher), the pupil proceeds to use it when confronted with fractions like $\frac{3a + 3b}{2a + 2b}$ and obtains $\frac{6}{4}$ as his answer (which is correct), and again uses the same rule on $\frac{3a + 3b}{a + b}$ wherein his answer, 6, is not so good. To avoid such habits the writer has found it useful to study the general subject of long division immediately preceding the work on fractions, and to work with exercises like: Divide $6x - 8$ by $3x - 2$, wherein the dividends *appear* to the pupil to be exactly divisible by the divisors, but are not. In the work on fractions the pupil is then occasionally asked to check his work by doing the long division.

There is still a third habit to consider. To the nine illustrations which Mr. Grossman has given on page 108, should be added one like

$$\frac{3\,(a + b) + x(a + b)}{5(a + b)}$$

Here each $(a + b)$ is a factor; and the pupil who has grasped Mr. Grossman's idea of an "uninterrupted succession of factors" will know that he can not cancel the factors as they stand. If, however, the inventive pupil (and it is this type of pupil who convinces the other pupils that the teacher has a horrid preference for working all problems in the most difficult way) does draw a line through the $(a + b)$s, he obtains the correct answer. Further, he can easily prove that his method will always be correct. He quickly explains that if he completed the factoring in the numerator so as to have an uninterrupted succession of factors, the remaining factor would be $(3 + x)$. Hence, since this is obvious, why not write the answer at once without doing the intervening work? Or, he may argue that the numerator consists of two terms, each of which was divided by the divisor *just as he was taught to do* when learning about monomial factors in a previous chapter. How this bright pupil may be answered will be seen shortly.

In attempting to dodge the pitfalls mentioned above, the writer has developed the notion of cancellation in two steps:

1. A fraction is simplified by dividing its numerator and denominator by some quantity.

2. To find a suitable divisor, factor the numerator and denominator. (Note that this step does not say that we factor because "factors, and not terms, may be canceled." We factor in order to find a suitable divisor.)

There is nothing new about step 1. The traditional method begins the work by showing that subtraction from or addition to a numerator and denominator changes the value of a fraction. Besides illustrating the idea by arithmetic fractions the traditional method also shows that omitting the 6s in $\frac{a-6}{b-6}$ is equivalent to adding 6 to the numerator and denominator. Here, too, we should be careful not to say "canceling the 6s is incorrect. . . " but "omitting the 6s is incorrect."

After finishing step 1, we conclude that we may only divide the numerator and denominator by some quantity. Traditionally, the next step has always been to show by arithmetic illustrations that we may cancel factors, but may not cancel terms. Hence we factor. While this is true, the writer does not believe that this distinction between factors and terms is the best way of *introducing* the idea, or that this explanation tells the entire story.

After insisting that the numerator and denominator must be divided, it seems more natural to ask next: "By what shall we divide?" The writer prefers therefore to begin by writing a fraction like $\frac{18x^2+9x-2}{3x^2+2x}$ on the blackboard and asking: "By what can we divide $18x^2 + 9x - 2$?" Evidently not by x, nor by y, nor 2, nor 3. The desired answer can be developed by a series of exercises like the following:

1. What is the quotient when
   (a) $(2x-5)\ (3x+4)$ is divided by $(2x-5)$? by $(3x+4)$?
   (b) $x(2x+1)\ (3x-2)$ is divided by $(2x+1)$? by x? by $x(2x+1)$?

2. Which is easier and quicker:
   (a) To divide $x^2 - 7x + 12$ by $x - 4$, or
   (b) To divide $(x-4)\ (x-3)$ by $x - 4$?

3. First factor the dividend, and then state the quotient when
   (a) $x^2 - 7x + 10$ is divided by $x - 2$.
   (b) $x^2 + 5x + 6$ is divided by $x + 3$.

4. By what quantities is $x^2 + 5x + 6$ divisible?

These exercises develop the idea that if we factor a quantity we can easily see by what it is divisible. The conclusion may be stated:

We must divide both numerator and denominator. We find a suitable divisor by factoring the numerator and denominator. Or,

We factor, *not* particularly because factors alone may be cancelled but because factoring is the best way of finding a divisor. The writer admits he smiles with satisfaction when he hears the pupil say "In order to find a divisor, we factor." This treatment makes use of a much neglected idea: the quotient of any quantity by one of its factors consists of the other factors. This treatment also allows the bright pupil to draw a line through the three $(a + b)$s in the illustration discussed above because the essential idea is to *find a divisor*, and some pupils are able to select a divisor without factoring. The writer smiles with still greater satisfaction when the pupil says "Without factoring, I can see that $(a + b)$ is a divisor of both numerator and denominator."

A BRIEF LIST OF MATHEMATICAL BOOKS SUITABLE FOR LIBRARIES IN HIGH SCHOOLS AND NORMAL SCHOOLS[1]

NATURE OF THE BOOKS IN THIS LIST

The following brief list is prepared to meet the many requests which come to Teachers College for information as to books for the use of teachers of mathematics or suitable for libraries in high schools and normal schools. An effort has been made to select a few books which will be helpful to students and teachers in such schools, but it must be understood that numerous other works are eminently worthy of having a place in such a list. No books in foreign languages have been included, but the department of mathematics of Teachers College will gladly give information relating to such books to any who may inquire. Except in the field of applied mathematics, no books have been included which are, in the ordinary sense of the term, text-books for secondary schools. The prices given are subject to change, but are approximately correct.

ALGEBRA

BURNSIDE, W. S., and PANTON, A. W. *Theory of Equations.* 2 vols. Longmans. $5.00 each.

The best work in English for high school teachers to consult on this subject. It elucidates many points in elementary algebra.

CHRYSTAL, G. *Algebra.* 2 vols. Macmillan. $5.00 each.

A very valuable source of information, material, and inspiration for teachers and students. The most comprehensive algebra in English.

DEDEKIND, R. *Essays on Number.* Translated by Beman. Open Court. 75 cents.

This translation of a classical work on number is very valuable for the teacher of algebra because it sets forth clearly the nature of the irrational number.

FINE, H. B. *College Algebra.* Ginn. $2.40.

A short standard reference work for use where a longer treatise is not necessary.

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[1] This annotated list of books was prepared by the following members of the staff of the department of mathematics of Teachers College: Professors David Eugene Smith, Clifford R. Upton, and William David Reeve, and Mr. William E. Breckenridge and Mr. Jekuthiel Ginsburg. It is herewith reprinted from the March, 1925, number of Teachers College Record.

APPLIED MATHEMATICS

BALL, K. F., and WEST, M. E. *Household Arithmetic.* Lippincott. $1.50.

Helpful for the teacher of mathematics in a vocational high school.

BRECKENRIDGE, W. E. *The Slide Rule. A Self-Teaching Manual.* Keuffel and Esser. 50 cents.

This may be obtained with a slide rule or separately. The range of problems extends from arithmetic through trigonometry.

BRECKENRIDGE, W. E., MESEREAU, S. F., and MOORE, C. F. *Shop Problems in Mathematics.* Ginn. $1.40.

A book including problems in the field of carpentry and metal work, designed to correlate the work of the classroom with that of the departments of mechanic arts and science.

BRINTON, W. C. *Graphic Methods for Presenting Facts.* Engineering Magazine Co. $4.00.

An interesting presentation of the entire field of graphic work and very helpful as a book of reference.

BROOKMAN, T. A. *Applications of Algebra Dealing with Automobiles.* Scribner. 50 cents.

The work of a group of mathematics teachers in a California summer school.

BROOKMAN, T. A. *Family Expense Account.* Heath. $1.00.

A summary of the subject of household accounts, for classes in arithmetic.

DALE, R. B. *Arithmetic for Carpenters and Builders.* Wiley. $1.75.

Presents in simple form the arithmetic found in the daily work of the carpenter and builder. It also includes geometry for builders.

EDGERTON, E. I., and BARTHOLOMEW, W. E. *Business Mathematics.* Ronald Press. $2.00.

Helpful for the teacher of mathematics in a commercial high school and useful in showing the applications of algebra.

FINNEY, H. A., and WALTON, S. *Mathematics of Accounting and Investment.* Ronald Press. $4.00.

Helpful in commercial high school work and in showing how algebra is used in modern life.

GARDNER, M., and MURTLAND, C. *Industrial Arithmetic for Girls.* Heath. $1.00.

Contains numerous simple problems in arithmetic applied to millinery, sewing, and novelty work.

GEPHART, W. F. *Principles of Insurance.* 2 vols. Macmillan. $2.00 each.

Helpful for the teacher of mathematics in a commercial high school.

GLOVER, J. W. *Tables of Applied Mathematics in Finance, Insurance, and Statistics.* George Wahr, Ann Arbor, Mich. $4.50.

A practical work for students in this field.

JOHNSON, J. S. *Practical Shop Mechanics and Mathematics.* Wiley. $1.40.

Suited for use in trade schools.

KEAL, H. M., PHELPS, N. S., and LEONARD, C. J. *Technical Mathematics.* 3 vols. Wiley. Vol. I, $1.50; Vol. II, $1.75; Vol. III, $1.25.

A series of books for teaching the mathematics of industry on the unit plan.

KNOEPPEL, C. E. *Graphic Production Control.* Industrial Management, New York. $10.00.

An excellent source book for graphic work and algebraic applications. Very helpful for teachers in a commercial high school.

KOCH, E. H. *The Mathematics of Applied Electricity.* Wiley. $3.50.

A mine of information on the application of mathematics to electricity.

MARSH, H. W. *Industrial Mathematics.* $2.00. *Technical Algebra.* $2.50. *Technical Geometry.* $1.75. *Technical Trigonometry.* $1.75. Wiley.

A series containing valuable material on the mathematics of modern industries.

PEDDLE, J. B. *The Construction of Graphical Charts.* McGraw-Hill. $2.00.

A treatment of graphical methods of calculation.

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MIKAMI, Y. *The Development of Mathematics in China and Japan.* Teubner, Leipzig. About 18 Reichmarks.

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SMITH, D. E., and MIKAMI, Y. *History of Japanese Mathematics.* Open Court. $2.00.

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A scholarly treatment of the teaching of elementary and secondary mathematics. It has for some years been a standard work and has recently been revised.

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Teachers desiring to familiarize themselves with the theory and practice of educational measurements will be interested in the following more general references:

LINCOLN, E. A. *Beginnings in Educational Measurement.* Lippincott. $1.60.

A simple treatise for the beginner in educational measurement.

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A book which every high school and college teacher should own and frequently consult. It is the standard work upon this subject.

KEYSER, C. J. *The Human Worth of Rigorous Thinking.* Columbia University Press. $3.00.

A strong statement of the value of mathematics in the training of the individual.

MONROE, P. *Cyclopedia of Education.* 5 vols. Macmillan. $25.00.

Should be in every library of education. It contains numerous articles on mathematics.

MORITZ, R. E. *Memorabilia Mathematica* or *The Philomath's Quotation Book.* Macmillan. $4.00.

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Reorganization of Mathematics in Secondary Education, being the Report of the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America. Sent gratis upon receipt of twenty cents for postage. Address, The Dartmouth Press, Hanover, N. H.

A voluminous report of the studies made by this committee. It should be owned and read by every teacher of mathematics.

RIETZ, H. L. *Handbook of Mathematical Statistics.* Houghton, Mifflin. $4.00.

A technical and scholarly mathematical discussion.

SLOSSON, E. E. *Easy Lessons in Einstein.* Harcourt, Brace & Co. $1.35.

A book which a reader without any extended knowledge of mathematics can appreciate.

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WHITEHEAD, A. N. *Introduction to Mathematics.* Holt. $1.00.

A valuable work for any teacher to possess.

YOUNG, J. W. *Lectures on Fundamental Concepts of Algebra and Geometry.* Macmillan. $2.00.

A work that should be read by all teachers of algebra and geometry.

YOUNG, J. W. A. *Monographs on Modern Mathematics Relevant to the Elementary Field.* Second Edition, 1924. Longmans. $4.00.

This work meets a want long felt by teachers. It considers such questions as theory of parallels, fundamental concepts of geometry, the transcendence of "e," and the general nature of the algebraic equation.

CARUS MATHEMATICAL MONOGRAPHS

Under the auspices of the Mathematical Association of America there is now beginning the publication of a series of expository presentations of the best modern thoughts and researches in pure and applied mathematics. This publication was made possible through the generous assistance of Mrs. Mary Hegeler Carus and the monographs are issued at a nominal price. Teachers who wish to keep in touch with the progress of modern mathematics should avail themselves of the opportunity of acquiring the works as they appear. They are published by The Open Court Publishing Company of Chicago.

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The American Mathematical Monthly. Lancaster, Pa. $4.00 per year.

Besides articles of special interest to teachers of mathematics this journal contains a department devoted to problem solving. Official organ of the Mathematical Association of America. Valuable to teachers in normal schools, in high schools, and particularly in colleges. Contains reviews of current mathematical literature. Very helpful in arranging programs for mathematics clubs.

The Mathematics Teacher. Lincoln School, 425 West 123rd St., New York City. $2.00 per year. In clubs of 25 or more, $1.75 per year.

This journal is published eight times a year under the auspices of the National Council of Teachers of Mathematics. It contains numerous articles on the teaching of elementary and secondary mathematics and should be read by every teacher of mathematics interested in these fields.

School Science and Mathematics. Mount Morris, Illinois. $2.50 per year.

Each month several articles in this publication are devoted to questions relating to the teaching of high school mathematics. The journal also reviews new text-books and contains a problem department.

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REFLECTIONS ON FOURTH DIMENSION

By A. M. ALTIERI

For over two thousand years Euclid was the accepted authority for geometry. In fact, in England the terms "reading geometry" and "reading Euclid" were considered synonymous. The learned Greek mathematician founded his geometry upon a set of axioms whence he derived all his theorems. These axioms, or truths, he derived from his own experiences, being unable to prove any of them. There was one which subsequent mathematicians did not consider self-evident, and which they tried fruitlessly for centuries to prove. It is a long, involved declaration which is equivalent to the statement that "through a point outside a line, only one parallel can be drawn to the line," and is universally known as the "eleventh axiom of parallels."

Finally, in 1830, a Russian geometrician, Lobachevsky, decided to leave the well-trodden path made by Euclid, and to blaze a new trail through the inviting realm of theory and speculation. Instead of assuming that through a point outside a line one, and only one, parallel could be drawn to the line, he assumed that through that point *many* parallels could be drawn. Upon this assumption, he constructed a complete geometry, in which theorems were logically proved just as in the Euclidian. A few decades later, it was found that Lobachevsky's geometry was true for the pseudo-spherical surface. This is a saddle-shaped surface, curved both positively and negatively, although the principal curvature is negative.

A quarter of a century later, Riemann, a German professor, emulated the Russian's example and also forsook Euclid. He deserted the venerable Greek just where Lobachevsky did, at the eleventh axiom of parallels. His predecessor had considered that many parallels might be drawn to a line through an outside point; Riemann went to the opposite extreme and assumed *no* parallels through such a point. Then, just as Lobachevsky had done, he built a geometry upon his assumption. A dozen years

Editor's Note: At the time this paper was written the author was a pupil in the high school at Newton, Mass. Mr. C. H. Mergendahl, the head of the mathematics department states that A. M. Altieri "wrote this paper for Alpha Gamma Tau, the Mathematics Club, without any assistance from any teacher". J. R. C.

afterwards, in 1868, men discovered that Riemann's geometry was perfectly applicable to the sphere. Both Riemann's and Lobachevsky's geometries, though each quite self-consistent were inconsistent in many places with Euclidian geometry and each other. Moreover, they were thought inferior to Euclidian because people believed them to be unlike reality. Time proved, however, that each was true for a certain kind of surface. Therefore, the following state of affairs exists: two men, in different times and places and upon opposite ideas, disagreed with the established authority. Each, working upon a theory alone, with little assistance from his own experiences, created a geometry, many of whose theorems could not readily be conceived of. The passage of a few years showed that their theories were correct, that their geometries really were compatible with reality. Why, then, cannot time some day demonstrate the truth of that most widely talked of and generally misunderstood mathematical theory, the fourth dimension?

Every space of surface upon which a geometry is to be constructed must have what is called a "constant measure of curvature." This is to enable one to move about geometrical figures without changing their size or shape. Both planes and spheres have this common measure. A good ordinary example of a surface which does not have it is that of an egg. If a triangle were drawn upon the broad end of the egg, it would be impossible to transfer it to the narrow end without changing its shape or size in some way.

It is evident that the three dimensional space to which we are accustomed, in which we move about and live our lives, has a constant measure of curvature. Obviously, a man can go to any part of that space, from the top of the highest mountain to the middle of the widest sea, without changing or becoming in any way distorted.

The measure of curvature of the ordinary plane is called zero; that of a sphere is positive, while the pseudo-sphere is negative. Evidently, the measure of curvature of our space, to which for clearness I shall refer as the space of experience, must be either zero, positive, or negative. Mathematicians tell us that there is a means of ascertaining which of the three it is. This means is afforded by the theorem from Euclidian geometry which states that the sum of the angles of a triangle is two right angles. To

understand how this applies, two things must be remembered. First, on a sphere (positive measure), the sum of the angles of a triangle is greater than 180° or less than 540°. Second, on a pseudo-spherical surface (negative measure), the sum is less than 180°. Therefore, if upon measurement the scientists found that the sum of the angles of any triangle, even of the great interstellar triangles, was greater than 180 (or less), it would be positive proof that our space of experience has a positive (or negative) measure of curvature. As a matter of fact, they have asserted that as far as they can at the present tell, since their instruments of measurement and especially telescopes, are inexact, the measure of curvature of the space of experience is zero. That is, the space of experience corresponds to a plane.

If the scientists were mistaken, which is possible though not probable, and if the measure of the space of experience were positive, the following would be true: a point, moving in a straight (geodetic) line in the space of experience, would, after continuing for a length of time inconceivable to us, ultimately arrive at the point whence it started. Moreover, the backs of our heads would be the most distant background. This appears unbelievable to us, but consider an illustration. In the days of Homer, men believed that the earth was a flat disk surrounded by boundless oceans. Imagine some scholar going to the Homeric hero Ulysses and informing him that if he traveled forever on the earth in the same direction he would eventually come to his starting place from the opposite direction. Certain it is that Ulysses would have been just as incredulous as we are when a mathematician tells us that it is possible that a point traveling onward forever in space may some time reach its starting-place.

Let us now take up once more the trend of our original argument, that the fourth dimension is a perfectly permissible and possibly some day an extremely beneficial theory. Ages ago, when man first learned to count, he used only positive numbers. Probably not for many years did negative numbers come into existance. Negative numbers were man's first trial at mathematical theory. No man can picture to himself "minus three trees." We can realize, of course, that three trees have been removed from a garden, but we can not imagine three negative trees. Nevertheless, we had no difficulty in creating and learning to use negative numbers. From one point of view, there is not a

particle of difference between this state of affairs and the fourth dimension. We cannot imagine that, but neither can we imagine negative numbers. From the study of negative numbers mankind derived inculculable benefit. Upon it, directly or indirectly, are founded the sciences of algebra and trigonometry, as well as our knowledge of irrational and imaginary numbers. In like manner we might also profit from an intensive study of the fourth dimension.

During the many centuries after algebra and Euclidian geometry came into being, scientists gradually began to study them in close connection with one another. It was discovered that geometry clarified many algebraic truths, while a judicial use of algebra rendered many geometric theorems much less difficult. Eventually, some observing soul remarked that "a", "a^2", and "a^3" all had some meaning in geometry; what was the meaning of "a^4", of "a^5", of "a^n" asked the inquiring element. Why should geometry stop at the third, when algebra, closely allied to the other, was obliged to continue to an infinite power? The question was unanswerable. It was to satisfy this craving for knowledge that the theory of "n" dimensions was instituted. Of these "n" dimensions, we will, for simplicity's sake, speak only of the fourth.

The mathematician, in regards to the fourth dimenesion, does not try to associate it with familiar ideas. He makes an unsupported assumption, that there is a fourth dimension; then, through pure logic, aided only by his mathematical knowledge, he proceeds to prove a series of connected statements in an effort to extract facts from his theory. This is exactly what Riemann, the founder of spherical geometry, did; what Lobachevsky, of the pseudo-spherical geometry did; what the makers of negative numbers did. By their study of apparent non-realities the world has been given much.

Upon close examination it will be understood that we think of things, in our everyday life, in connection with three axes, each a line perpendicular to both the others at their point of intersection. A concise statement of the meaning of "the fourth dimension" might be that *the fourth dimension is the direction of a line which is perpendicular to every one of three such axes at their point of intersection.* This is obviously impossible, but it has already been stated that the fourth dimension is an assumption, and that

the man who studies it must abandon all familiar conceptions of space. At this point let me add that when studying the theory of "n" dimensions, one must revise his definition of space, using it in a broader sense.

Four-dimensional space has been exhaustively studied by analogy; while this method of reasoning is often dangerous and misleading, it nevertheless gives us some interesting lights on the subject. It can be shown that a hypersolid (name applied to a four-dimensional figure) revolves about a plane. This motion, of course, is inconceivable to us, but there again looms large the fact that we must proceed from reason alone, not from the evidence of our senses. Since, in two-dimensional space, a figure can rotate only about a point; since, in three-dimensional space, a body can rotate only about a line; it follows that in four-dimensional space a body rotates about a plane.

By analogy, some of the characteristics of a four dimensional body can be determined. If we move a point through the first dimension we get a line; if we move a line segment through the second dimension, we get polygon which, for clearness, we will assume to be a square; if we move the square through the third dimension we get a cube (or parallel opiped of some sort); therefore, if we move the cube through the fourth dimension, we will get a four dimensional figure. It can be shown, by elaborating this analogy that such a figure has eight cubes, sixteen points, twenty-four planes, and thirty-two lines.

The fourth dimension helps to explain the phenomenon of symmetrical figures. Two symmetrical polygons can not be made to coincide unless one is lifted out of the plane and turned over in three dimensional space. From this we deduce that two symmetrical figures can not be made to coincide unless one is passed into space next higher in dimensions, and turned over before being returned to the original space. Thus two symmetrical tetrahedrons could be made coincident by moving one into four-dimensional space and inverting it. The hands are good common examples of symmetrical objects, as are corresponding right and left parts of any animal. Symmetry furnishes a possible means of positively proving that the fourth dimension exists. The following shows this: there are two forms of tartaric acid, alike in every respect but that the crystals of one are symmetrical to those of the other, so that one reflects polarized light to the right

while the other reflects it to the left. One of these forms changes to the other without any apparent chemical change or demonstration of force; if this could be proved, it would be actual proof of the existence of the fourth dimension.

Many other things could be shown by analogy, among them that while in three-dimensional space three numbers determine a point, in four-dimensional space four numbers would be required. Moreover, since in a plane only three points can be placed, each equidistant from all the others; and since, in three-dimensional space, four points can be so placed; in four-dimensional space *five* points could be so arranged. Proceeding further, we find that a little thought will reveal that in ordinary space an object has six degrees of freedom; namely, three translations along and three rotations about three axes, while three fixed points stop all movement. A hypersolid, on the other hand, has ten degrees of freedom; namely, four translations along four axes and six rotations about six planes, while four fixed points stop all movement.

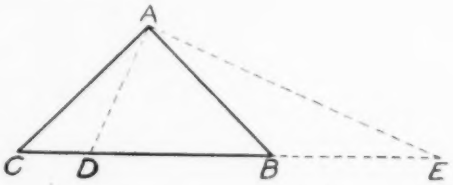
Turning away from the strictly mathematical end of the subject, we learn that men have proved that in hyperspace many apparently impossible things can be done; for example, a flexible sphere can be turned inside out without tearing it, the links of a chain can be separated without breaking them, and knots in a cord can be untied without removing the fastened ends.

In conclusion, it seems to me that the theory of four dimensions is quite logical and reasonable; and that it may some day add materially to man's store of knowledge, as I have endeavored in the first half of this essay to show.

PROOF OF THE THEOREM OF PYTHAGORAS

By ALVIN KNOER
(A Milwaukee High School Pupil)

The square on the hypotenuse is equal to the sum of the squares on the other two sides.



Given right $\triangle ABC$ in which $\angle CAB$ is a right angle.

To prove: $\overline{CB}^2 = \overline{AC}^2 + \overline{AB}^2$

Proof: (1) Lay off BD and BE on line CE equal to AB. Draw AD and AE.

(2) In $\triangle ADC$ and $\triangle AEC$, $\angle ACD = \angle ACD$. (By identity).

(3) Since $\triangle ADB$ is an isosceles $\triangle$ by construction, $\angle ADB = \angle DAB$. (Base angles of an isosceles $\angle$ are equal.)

(4) Similarly $\angle BAE = \angle BEA$.

(5) $\therefore \angle DAB + \angle BAE = \angle ADB + \angle BEA$. (Addition axiom.)

(6) $\therefore \angle DAE = \angle ADB + \angle BEA$... (Subst. axiom.)

(7) $\angle DAE + \angle ADB + \angle BEA =$ a straight $\angle$. (Sum of int. $\measuredangle$ of a $\triangle$.)

(8) $\therefore$ $\angle DAE$ is a right $\angle$. (Since $\angle$ DAE is equal to the other 2 $\measuredangle$ of the $\triangle$ it is equal to ½ of a straight $\angle$.)

(9) $\angle CAD = \angle CAE -$ ① right $\angle$.

(10) $\angle BAE = \angle CAE - 1$ right $\angle$.

(11) $\therefore \angle CAD = \angle BAE$. (Things equal to the same thing are equal to each other.)

(12) $\therefore \angle CAD = \angle BEA$. (Same reason.)

(13) $\therefore \triangle ACD \sim \triangle ACE$. (If 2 $\triangle$ have 2 $\measuredangle$ of the other, the $\triangle$ are similar.)

(14) $\therefore CE/CA = CA/CD$. (Homologous sides of similar triangles are proportional.)

(15) But since $CE = CB + AB$ and since $CD = CB - AB$, $(CB + AB)/CA = CA/(CB - AB)$. Any quantity may be substituted for its equal.)

(16 $\therefore \overline{AC}^2 = \overline{CB}^2 - \overline{AB}^2$. (The product of the means is equal to the product of the extremes.)

(17) $\therefore \overline{CB}^2 = \overline{AC}^2 + \overline{AB}^2$. (Subst. axiom.)

Therefore, the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

The manner in which I happened to write this proof is not as one would commonly suppose. It was during my study of Geometry II at Washington High School that the idea was first suggested to me at all. Mr. Ericson, teacher of mathematics, told us at the time we were studying the proposition, that if any of the students were successful in working out a new proof for the Pythagorean theorem, he would see to it that the matter would be properly advertised in the local newspapers.

Although I never expected to be able to develop a new proof, I tried a little experimenting a few days later just to pass time. This proved so interesting that I continued working out all kinds of ideas. The usual result of fifteen or twenty minutes of work, however, was that the solutions would boil down to an identity, thus bringing me back to the original starting point.

After experimenting in this manner for about a month I began to realize that my efforts were useless. Finally I struck upon a plan which seemed to me to be more logical than others because instead of starting from the beginning I started from the end and took the fact that the square on the hypotenuse is equal to the sum of the squares on the other two sides for granted.

Then I broke this equation up into several other equations and finally ended up in a proportion. We had just finished similar triangles so the proportion which I had developed suggested the usage of similar triangles. Then after I had the triangles constructed it was a matter of only an hour or so to work out the proof. In this way I had a definite problem of proving two triangles similar thus making the problem less difficult.

DISCUSSION

DISCUSSION—The excellent paper in your February issue by Horace C. Wright, "Some True-false Examinations for Use in General Mathematics" has come to my attention.

I am not an admirer of this type of examination. Most of my objections arise from my personality, are matters of opinion subject to dispute and perhaps not valid. There is one defect that I believe is insuperable. I refer to the fact that unqualified statements are not of necessity either true or false. Such an examination becomes in considerable degree a measure of the ability to read the examiner's mind.

Permit me to illustrate my point from the article in question, which is rather freer, than the average of this type of examination, from ambiguous questions.

Statements 1 and 2 on page 88 are not a fair test. The absence of sufficient data to determine geometrically the truth of the statement does not, of itself, make the statement untrue, the only basis under the circumstances is comparison of the figures themselves. The pupil with both knowledge and sense will use this basis in answering, the pupil with knowledge alone will answer "wrong" on account of insufficient data, the pupil with neither knowledge nor sense will guess. Note that the question does not involve proof but truth of conclusion.

Statement 3 seems either trivial or to conceal a "catch." A knowledge of the boy's usual reaction to command might be involved. All three of this group of questions relate to matters of opinion not provable by the given data even when the drawings are present.

A number of the elements in the test starting on page 89 seem to me objectionable. I give them with the true-false notation as they appear in that paper.

"False, 7. The line from Chicago to New York to New Orleans is a curved line." To which of the infinity of lines connecting these points does the examiner refer? Let us not forget, as perhaps the student who thought the statement true did not forget, that three points not in the same straight line determine a circle.

"False, 15. A circle is bounded by points." Why false? and how prove this falsity to the pupil in view of the locus idea?

"False, 20. A sheet of squared paper is just as long as it is wide." Depends upon the sheet, not upon inherent property of squared paper.

"True, 32. The string around the outer surface of a new, blown-up bicycle tire forms a circle." This depends upon in what manner or direction the string is wound. This is certainly a case of guessing what the examiner is thinking of.

"True, 38. When two radii form an angle it is a central angle." False unless they are radii of the same circle. Should a pupil be penalized for knowing this simple fact?

"False, 40. Compasses measure angles." If the objection is to the use of the term measure, instead of are used to measure, the question is unfair. If on the other hand the pupil is to guess that the examiner is thinking of a draughtsman's compass and not a surveyor's compass, the question is worse than unfair.

"True, 41. The degree is the unit in angular measure." Not true till the is changed to a.

"True, 42. The scale on the protractor runs from 0 degrees to 180 degrees." Not true of all proctractors, not ever generally true, and of no moment if it were.

"True, 43. Angles may be measured inside or outside of doors." How does one get inside of a door to measure an angle? The pupil who so interprets the question answers "false" and is unjustly marked wrong.

"False, 46. Addition and subtraction of angles are performed with pencil and paper." Why false? I deal with angles constantly and so manipulate them. This is another test in guessing what the examiner is thinking of.

"False, 58. The diamond on a playing card is equiangular." Seems like a knowledge of playing cards rather than of geometry. Many people object to playing cards. Should a child be penalized for the religious belief of its parents?

"True, 59. A runner running the bases in a ball game runs around a rhombus." Answer should be false as a baseball diamond is a square not a rhombus.

"False, 61. If two angles have their sides parallel left to right and right to left the angles are equal." This seems like rather confused English.

Thirteen out of 82 questions are subject to serious question as to exact meaning or correctness of given answers. Enough to put the good student in the dullard class.

Teachers are overworked and pressed for time. It is not possible in the time they have available to make 50 or 100 statements that relate to significant facts, that are so clearly and exactly worded that they can be marked as either true or false without restriction of the answer or emendation of the question. Ambiguity is worse in this type of question than in the old, for in the latter the answer often shows that the question was not understood by examiner and examinee in the same sense.

EDWARD TAYLOR,
Pomona College, Cal.

NEWS NOTES

The Detroit Teachers Mathematics Club has announced the following program for its six meetings to be held during the current year.

I. What Should be Included in The Mathematics for Intermediate Schools? by J. V. McNally, Principal Sherrard Intermediate School.

II. Differential Courses in Mathematics For Pupils of Differing Abilities, by I. M. De Voe, Head of Department of Mathematics, Highland Park School.

III. The Relation of Mathematics to the Sciences, by J. E. Tanis, Principal Northern High School.

IV. Aims and Methods in Mathematics in the Secondary Schools, by Alfred T. DeLury, University of Toronto.

V. Dynamic Symmetry, or the Relation of Mathematics to Art (Illustrated), by Miss Marie Gugle, Assistant Superintendent of Schools, Columbus, Ohio.

IV. Reports on Experimental Work in Mathematics in the Detroit Schools.

The officers of the club are: Miss Orpha E. Worden, President; Mr. H. Yutzey, Vice-President; Miss Paula Henze, Secretary; Mr. W. B. Templeton, Treasurer; Miss Katharine Hine, Chairman of Social Committee; Miss Sadie Alley, Chairman of Membership Committee; Mr. J. V. McNally, Chairman of Program Committee.

Mr. Harry C. Barber, formerly of the English High School, Boston has recently been appointed head of the Mathematics Department of the Charlestown High School.

The regular Fall meeting of the Philadelphia Section of the Teachers of Mathematics of the Middle States and Maryland was held Thursday, October 15th, 1925, at 3:30 P. M. in the William Penn High School for Girls. The president, Mr. Brecht, presided. The minutes of the March meeting were read and approved.

The section had as its guest, Professor John H. Pitman of Swarthmore College. Professor Pitman gave an interesting address on "Observing a Total Eclipse of the Sun—The Problems,

Instruments and Results." This was illustrated by Motion Pictures.

Professor Pitman's address was followed by a short business meeting. The question of a dinner to be held later in the year was discussed. A majority of those present were in favor of a dinner.

David Eugene Smith, John A. Foberg and William D. Reeve have recently published with Ginn & Company a text book in *General Mathematics* for use in the first year of the high school.

At the Spring Meeting of the Association of Teachers of Mathematics in New England last May, the following vote was proposed and referred to the Council for action. The Council at its meeting on October 25th, 1925, passed the vote.

Moved, that the Council of the Association of Teachers of Mathematics in New England heartily approves the recent changes in the Requirements of the College Entrance Examination Board which decreases the number of book theorems in Geometry and which lessen the emphasis on complicated algebraic manipulation.

This shall not be considered as an approval or a disapproval of any particular examination.

Also moved, that a copy of this vote be sent to the Mathematics Teacher and to the Secretary of the College Entrance Examination Board.

The Iowa Association of Mathematics Teachers held the following program at Des Moines on November 6.

I. Classroom Methods in Algebra, J. A. Nyberg, Hyde Park High School, Chicago.

II. Guiding Principles in the Teaching of Mathematics in Junior and Senior High Schools. Raleigh Schorling, Ann Arbor, Michigan.

NEW BOOKS

A Tentative List of Objectives in the Teaching of Junior High School Mathematics With Investigations for the Determining of Their Validity. By Raleigh Schorling. George Wahr, Publisher, Ann Arbor, Michigan. Pp. 137.

In this valuable contribution to mathematical literature, Mr. Schorling has as his aims, "(1) to review and summarize the studies made by other workers in the field, (2) to advance the line by reports on special problems bearing on the general topic, (3) to construct a list of specific objectives using five criteria, (4) to present a more complete picture of the strength of mental connections in arithmetic for pupils at the beginning of the seventh grade, and (5) to formulate a technique whereby materials written for children may be adjusted on the basis of classroom trial under widely diversified conditions.

The author outlines his procedure as follows:

I. The formulation of a brief list of guiding principles implying a philosophy of secondary education.

II. The selection of specific objectives on the basis of the following criteria:

(a) A summary of the elements for which some kind of a positive case can now be made by employing one or more objective studies dealing with the social uses of mathematics.

(b) Practice as determined by an inventory of selected courses of study.

(c) Report of the National Committee on Mathematical Requirements.

(d) An inventory of the first seven series of mathematical texts written for the junior high school grades.

(e) A highly selected jury of five educators especially interested in the junior high school and five leaders in the teaching of high school mathematics.

III. The refinement of the elements selected and their grade placement by means of an extensive classroom trial under test and report conditions.

In Chapter I the guiding principles are presented:

I. For Secondary Education
   (a) human satisfactions
   (b) social progress
   (c) habit formation
   (d) method
   (e) individual differences
   (f) measurement

II. For the Junior High School
   (a) Minimum essentials
   (b) standards
   (c) exploration
   (d) present values
   (e) specific objectives
   (f) practice in citizenship

III. For Junior High School Mathematics
   (a) immediate values
   (b) psychological organization
   (c) content
   (d) the unifying principle
   (e) mastery
   (f) social utility

Here we have presented, in condensed form, the basic principles for the organization of the intermediate grades, and also of the curricula of these courses. It is fair to say that not all these objectives are being observed in some junior high schools of the country today. What we can see, however, is definite progress —there is abundant evidence of this—progress in organization, in curriculum revision, in forward looking methods of teaching.

This book will be of material assistance in directing scientifically the progress development of the mathematics of the junior high school. It is hoped that Mr. Schorling's effort may stimulate similar studies in other subjects of the curriculum.

The author reports the results secured by giving an inventory test to 3,260 pupils beginning grade. Pupils were tested in 125 simple elements of arithmetic. Of the 125 elements, there were only eight in which 90% of the pupils were proficient, and in five tasks less than 10% of the pupils were successful. The tremendous task of the teacher of junior high school mathematics is apparent. Objective studies relating to the choice of materials

in the mathematics of grades seven, eight and nine present data justifying unmistakably such a program as outlined by the National Committee on Mathematical requirements.

The results obtained from a jury of mathematical leaders and general educators seem to be less convincing. Perhaps there are involved here some of the fallacies of the questionnaire method of investigation. A study of this type, including nice gradations of opinion on widely varying and numerous elements, produces results not too impressive from a scientific standpoint.

In this book Mr. Schorling has again made a contribution. He has made many—all of real educational value. The mathematical world appreciates such a carefully organized presentation, and it is hoped that the material included will be carefully studied by all teachers in the field of Junior and Senior high school mathematics. A close analysis of the contents will do much, not only to serve as a guide for teachers in the intermediate grades, but also to provide a basis for the solution of the many problems relating to the better articulation of the mathematics of the elementary grades, the junior and the senior high school.

WALTER F. DOWNEY, English High School, Boston, Mass.

Mathematik und Malerei. By Dr. Georg Wolff. Zweite Auflage, Leipzig, Teubner, 1925. Pp. 85. Price 2 marks.

To one who sees in mathematics something beyond college entrance requirements or syllabi prepared by the city or state, this little work by Dr. Wolff will be a delight. The book first made its appearance in the first year of the World War, at a time when few people were thinking of the finer aspects of life; and now, after ten years, it is issued in revised form and should at last have the recognition which it merits. It is one of the well-known series, the Mathematisch-Physikalische Bibliothek, the entire list of which is worth the careful persual of all who teach either of the related sciences for which it stands.

The work is divided into two parts, (1) General discussion of perspective as used in painting, and (2) the applications of mathematics in pictorial art. It considers such topics as the dependence of a picture upon color, atmosphere perspective, and linear perspective; the mathematical basis of perspective; the application of the laws of perspective to architectural drawing, with numerous references to the historical development of this

phase of art; and the relation of mathematics to portraiture. Throughout all the discussion the historical element is emphasized, the text being illustrated by thirty-four reproductions of works by such masters as Durer, Leonardo da Vinci, Giotto, Titian, and Botticelli, and by half-tones of Greek vases, Popmeian wall paintings, and paleolithic art. Many of these illustrations are more fully explained by superposed lines to show the laws of perspective followed by the artists. To the teacher of mathematics the frontispiece will be particularly interesting, it being the portrait of Pacioli by Jacopo de Barbari.

Dr. Wolff is Studiendirektor in Hanover and is widely and favorably known for his work on mathematics as taught in English schools and for his editorial connection with the Unterrichtsblätter für Mathematik und Naturwissenschaften.

David Eugene Smith

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(To be Continued)

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ANNUAL MEETING OF THE NATIONAL COUNCIL OF MATHEMATICS TEACHERS

The next meeting of the National Council of Mathematics Teachers will be held at the Raleigh Hotel, 12th Street and Pennsylvania Ave., N. W., Washington, D. C. on February 20th, 1926. There will be programs both in the morning and in the afternoon. Also there will be a dinner at the Raleigh Hotel, at a cost of $2.50 per plate. The officers are making every effort to reduce this price, but this may not be possible as the hotel is making no charge for the rooms for the meetings of the Council. Reservations for the banquet should be sent directly to Mr. Harry English, the Chairman of the local committee: Office of the Board of Examiners, Franklin School Building, Washington, D. C.

Raleigh Schorling, President

BALLOT
ANNUAL ELECTION OF OFFICERS
of the
National Council of Teachers of Mathematics

According to motion passed at the Cincinnati meeting, the Executive Committee is making the following nominations for officers of the National Council of Teachers of Mathematics.

PRESIDENT—(Vote for one)

☐ William Betz, Rochester, New York
☐ Marie Gugle, Columbus, Ohio

VICE-PRESIDENT—(Vote for one)

☐ Walter W. Hart, Madison, Wisconsin
☐ E. H. Taylor, Charleston, Illinois

EXECUTIVE COMMITTEE—(Vote for two)

☐ W. C. Eells, Walla Walla, Washington
☐ E. B. Lytle, Champaign, Illinois
☐ W. D. Reeve, New York City
☐ F. Touton, Los Angeles, California

DIRECTIONS FOR VOTING—

Place a cross in the square before each name of your choice. Each member of the Council (or reader of this journal) should mark the ballot as suggested, tear out and mail to Secretary-Treasurer, J. A. Foberg, Camp Hill, Pennsylvania before February 1st, 1926. Mr. Foberg is instructed to bring the ballots to the annual meeting to be counted by the Executive Committee.

In case a reader does not wish to mutilate this magazine, a copy may be made of the ballot on an ordinary sheet of paper which, if clearly marked so as to indicate the voter's choice, will be counted by the Executive Committee.

LEVIATHAN

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